The Open University

Exploring mathematics: maths in nature and art

The practical application of fractals

Prof. Ian Stewart, University of Warwick
Different mathematicians have very different attitudes to research. Some of them do it simply because there's an interesting problem, they can do their mathematics, they want to know what it's like, and they can get involved in the problem for its own sake. Others, at the other end of the spectrum, will only work on a problem if they can see real practical pay off.

Dr Michael Barnsley, Iterated Systems
It's almost like it would be strange to be a poet and never read your poems to anyone.

V/O Francesca Hunt
Michael Barnsley is an English mathematician who now runs a multi-million dollar company in Atlanta, Georgia. He exploits fractals, simple equations producing complicated pictures, by turning this idea on its head.

Prof. Ian Stewart
Well some people hope that just as here we see complex behaviour generated by very simple rules, and as in fractals we see complex pictures generated by very simple rules, that it would be possible to take a very complicated picture and work backwards to the simple rules that capture that complexity.

Dr Michael Barnsley
Well the problem is that if you take a single picture and you put it in a computer, it occupies an awful lot of space. For example, the amount of pictures required for one second of video would take the same space as one whole fat telephone book with two thousand pages. So you've really got a problem if you want to send video over 'phone lines, for example, just imagine how many telephone books, you'd have to send sixty telephone books-worth every minute, and if you've ever tried faxing documents over 'phone lines, it goes much slower than a telephone book every second. So there's a problem, and the problem is instead of sending all of that space you just want to send a little bit of it, that sort of amount, so that you could reconstruct the whole telephone book from the little bit of space. Or to put it another way, you could reconstruct each picture from a little bit of information, you could recreate the picture back again, and the way we do that is using the theory of chaos and fractals. Now over here we've got a picture, guess of whom? Henri Poincaré and what we've done is we've already digitised him, and we've set the fractal compression system to work on the picture. David, can you show us how this picture is broken up like this.

David Rand
So the fractal compressor has split this image into many thousands of small squares. For each of those small bits it's found a bigger bit elsewhere in the picture that it can transform and make look like a small bit. The compression comes by sending only the transformations between the bits, and not the information that's in the bits themselves. This sort of gives a recipe for a way to make Poincaré

V/O Francesca Hunt
To get an idea of the fractal compression method, look at the bridge of Poincaré's nose. If we break it up into smaller squares, then it turns out that there are only four key patterns. Now we search for larger pieces of the picture, which are similar to our small key patterns. For example, this large square rotated through ninety degrees, shrunk and moved over, matches this smaller square. In fact, every small piece of this picture can be matched on to one of the four larger pieces by these transformations.

David Rand
Ah, here's an example of a block in the corner of one of the postcards in the background that matches with a bit of his coat. The interesting thing here is that this is really quite a dark square, and the darker part is on the bottom, but in the red square it's a much lighter square, and the darker part is on the right side, so the green square has been rotated, and then it's had its brightness and contrast adjusted, the same way you adjust the knobs on a television set, so that although this is quite dark its variation is the same as the lighter variation that you see here, and it's been twisted through a hundred and eighty degrees. Here's a place where the computer has made a match between a bit of Poincaré's hair and a distorted piece of his wallpaper, and this green square will be distorted and mapped into the red square you see on his head.

V/O Francesca Hunt
The remarkable thing is that you only need to have a list of those transformations in order to reconstruct the whole picture. If you apply their equations repeatedly, then after just a few iterations the picture's indistinguishable from the original.

Dr Michael Barnsley
So a CD-ROM only holds about six hundred megabytes of data which means that if it was not compressed it'd only hold a few minutes at most of video. So what happened was Microsoft wanted to put numerous pictures, colour photographs, onto a multi media encyclopaedia called Encarta. They actually compressed seven thousand colour photographs, all as fractals. No-one knows it when they look at the pictures, they just think they look like photographs. They're really looking at fractals. Whatever you see anywhere, any time, any place ever, you will see more of it in that same picture, and that each picture is fundamentally different. I can see in front of me an orange rock and, goodness me, I never sat here before, I never looked at this rock, but big pieces of it look like smaller pieces. Again, I look and I see the trees behind me and there's not a little strange, isolated, existential leaf in the air all on its own, there are tens of thousands of leaves, and they're all of different sizes. It's as though they are trying to obey a rule or a law of nature, the rule of affine redundancy and real world images. It's that affine repetition that we exploit by the mathematical theorem that is used in fractal image compression. It counts on the fact that in the picture you are going to, whatever is in a picture, there is going to be more of it at different scales. And the strange thing is that's exactly the property that mathematical fractals have. So if I pretend that what I see out here was a giant fractal, and I just forget that it's the real world for a moment, and I say as a mathematician, how would I find the formula for that fractal, well that's the problem that we solved.

Dr Phil Rippon, Open University
It's great to see this area of pure mathematics which has been going for almost a hundred years, really without any view to applications, unexpectedly producing an application which might affect everybody's lives.

Prof. Caroline Series, University of Warwick
I think that to predict the course of mathematical research you would have to be some sort of genius, even better than Poincaré. In other words, I think it's a jolly difficult thing to do. In fact, the most unexpected things that seem at the time quite useless turn out to be absolutely fundamental.

Prof. Ian Stewart
Well some of the things that have clearly influenced the development of mathematics over the last century have been computers, visual imagery, and interactions with nature, but much more important than that, the rogue element is human imagination, and it's that which is completely unpredictable and that's what makes research so exciting.