Exploring mathematics: maths in nature and art

How to create a spiral

Human designers often take their inspiration from nature, and designs based on spirals are common.
A colleague showed me a very nice spiral you can make using rotations and scannings.

The idea is to start by drawing a square, and on top of that you draw a right angled triangle, which is isosceles. So this angle is Pi by four.

That's the basic shape, the basic figure. Next you draw exactly the same shape but scaled down by a factor on this side of the triangle. So there's a square, and then that's followed by a right angled triangle. Now you can see that this second shape is scaled down by a factor which is in fact one over root two. If that's one, then that's one over root two.

And then you repeat the process, on this side you draw a square, and a right angled triangle, and then a square and a right angled triangle. You can see that this will produce a spiral, but it's not clear immediately what the centre of the spiral will be, but if you investigate it, you find that the centre of the spiral is going to be the point that you obtained by drawing these lines.

So the spiral's obtained by taking a basic figure and repeatedly rotating it through Pi by four, about this point, and scaling down by a factor one over root two.

Well you can get a much more accurate picture of what happens in the long term by using MathCad.

Here's a MathCad document that draws that spiral, we've built it up in stages. First we define the rotation angle, that's theta Pi by four, forty five degrees, and the scale factor, which is A equals 1 over root 2 about 0.7.

NEXT WE DEFINE THE ROTATION MATRIX IN THE USUAL WAY R OF THETA, AND A UNIFORM SCALING MATRIX U OF A, DEFINED LIKE THAT.

Now if we multiply U of A and R of theta together, this gives us a matrix A which has those entries, and that matrix will perform both the rotation and the scaling together.

The next thing you have to do is to define the vertices of the basic figure. The vertices are defined as vectors, columns, and the first few of these defined the vertices of the square and the last few defined the vertices of the triangle on top of the square.

And if you plot the basic figure, this is what you get, and if we want to plot the successive images of the basic figure, we're going to have to multiply the co-ordinates of the vertices by powers of A. A, A squared, A cubed and so on, and that's what happens here.

So if we change to one we get one more. Each time we increase capital N we get one further successive image.

By the time you've plotted eight of those you can see that the spiral has performed a complete turn around the centre which is the origin.

Well if you look at the space that's within this spiral, then it turns out that you can fit in several copies of the original spiral into that space. And we've done that in another MathCad document.

So in this document we're plotting essentially exactly the same spiral, but instead of having one basic figure to start with we have four of them, one blue, one magenta, one black, and one red. And we've plotted twenty iterates of each of these four basic shapes, and you can see that if you follow the blue spiral round, then after eight iterates it exactly fits inside the beginning of the red spiral.

So these four spirals will in fact cover the plain if you extend them forwards and backwards, that is they tessellate. Now you can see that more clearly by zooming in on the diagram by changing the scale, here I'm changing it from two to and eighth, it appears to be filling in the plain, and that the diagram is symmetric in fact it's very striking.

If you wanted to use the design you could copy it out of MathCad and put it into a paint package.

Now, if I want to I can colour that in very many ways, using flood fill, for example you could pick this green colour and fill that square with it, and you might fill other squares with the same colour, you might try orange.

Well there are endless possibilities for such colour schemes, and this design may even be of interest commercially.

For example, carpet designers need patterns that fill the plain.