Exploring mathematics: maths in nature and art

Slicing cones

John Fauvel

In this part of the video we'll be exploring different representations of conic sections. They are the curves generated by slicing the cone by a plane.

Let's concentrate on the conic section given by slicing the cone by a plane parallel to one of its sides.

How might we represent this slice through a cone as an algebraic equation?

We first need a coordinate system to relate things to.

Often a property of a curve is so characteristic that it can be used as a definition. The parabola can be thought of as the set of points equidistant from a fixed point. The focus and a fixed straight line - the directrix.

Each point on the parabola is the same distance from the directrix - measured along the shortest way, which is the perpendicular to it - as it is from the focus. This is what characterises the parabola.

This relationship between the focus, the directrix and the parabola, is independent of any coordinate background. However in practice, if we are to be able to talk about properties of the curve, we often need to refer to an underlying coordinate system. And to keep the equations relating the curve to the coordinate simple, its often convenient to use the so-called *standard* position for the axes.

All the different representations that we've seen for the parabola can be explored in much the same way for each of the other conic sections. For example, by looking at the ellipse as a slice of a cone.

The ellipse has a focus-directrix property. A point on an ellipse is always nearer to the focus than to its directrix in a ratio which is a constant from any particular ellipse. As for the parabola, the locus of such points can be taken as the basis for any defining representation.

Ellipses can be represented parametrically, and the same pattern of different representations is also true for the hyperbola.

These strong parallels between the different conic sections and their representations are what has led to their being called the 'family' of conics. kinds of transformation have been used to repeat them?