Exploring mathematics: maths in nature and art

Where art meets maths

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The decorative designs on these fabrics are different in many ways.
Not just in size and colour but also in the shapes that they make.

But there are also many similarities between them.
In particular the ways the designs are composed have a lot in common.

Decorative designs can be found in the earliest artefacts...

And in every culture.

This tent hanging was made in this century, as were these Mud cloths.

But this South American material is over two thousand years old.

In creating new designs, it helps to appreciate both what is possible, and the limitations.
That's where art and design meet mathematics.

In seeking ways to create new designs, it's useful to characterise the properties of existing ones - to describe them mathematically and to find a way of classifying them.

All the designs we're considering are based on the idea of repetition, of ways of repeating some basic motif.

For simplicity's sake on this video band, we'll focus mainly on shapes and largely ignore colour.

This material is over 500 years old and has this as a motif. To construct the design, the basic motif is simply translated, horizontally.

There's translation, too, in this design - of the blue leaf motif.

Another way of generating designs is by reflection.

This simple design is formed by first reflecting the basic motif - and then translating it.

So reflection and translation are two ways of generating designs. But the generation of designs often involves another common transformation - namely rotation.

Here the blue and black motif has been rotated - and there's also rotation in the composition of other parts of the design.

The rotation is specified by a centre of rotation and an angle.

Now the designs on this material include the vertical translation of a diamond shaped motif.
But elsewhere the fabric design contains other transformations.

Here a translation on its own won't work, because the two leaf shaped motifs are of opposite orientations. So how could we get a copy of one from the other?

Well, one way is first to translate vertically and then reflect. But another way might be to reflect first and then translate.

A glide reflection is a reflection followed by a translation or vice versa.

In fact there are always several ways to achieve the same overall transformation.

Let's have another look at the blue leaf pattern. To turn the corner, the blue motif first has to be rotated through an angle of p/2 and then translated vertically.

But by ignoring the colour of the leaf and considering only its shape, more of the border could be generated from the basic outline - by reflection, rotation and translation.

In fact, there often isn't a single "simplest" motif. Rather there are lots of different ways of choosing a simple bit of a design to repeat.

Even this could be seen as a single bird, reflected in water.

So are all designs created by some combination of reflection, rotation and translation?

Well that is true here, because we're limiting ourselves to look only at those designs which repeat a single motif in some regular way and where the generating motif stays the same size.

Mathematicians call transformations which preserve lengths isometrics, from the Greek "iso" meaning "the same" and "metric" meaning "measurement of distance"

This design might seem fairly complicated but what do you see as repeating motifs, and how are your chosen motifs repeated.

For the moment don't choose anything too small. Just look for something obvious. Something that has clearly been rotated or translated, or both, all over the fabric.

One choice you might have made is this blue and black one.

Indeed if you ignore the colours, the overall pattern can be generated by successive rotations of this blue and black motif.

But you may have spotted other repeated motifs. There's this six pointed flower shape for instance.
And this one, three orange diamonds joined by a white blob.

Both the six pointed flower and the three orange diamonds have been copied all over the fabric by translations and rotations.

So instead of being generated by the blue and black motif, the whole pattern could instead be thought of as generated by a combination of two motifs, the flower one and the three diamond one.

But can you see how these three large motifs could be thought of as the combination of smaller ones?
Well the blue and black one could be divided into four. So the overall single blue and black motif is actually the result of reflecting and rotating a quarter of itself.

The flower can be generated by rotating each of its petals. But each petal is also the reflection of half a petal.

If we ignore the white blob, the three-diamond motif is made from rotating a single diamond - but even that can be made from reflecting smaller triangles.

There are only four possible isometric transformations -
....reflection...
....translation...
....rotation...
....and glide reflection.
Let's end this video by looking at even more designs. Again, what do you think are the basic motifs? And which of the four kinds of transformation have been used to repeat them?