The Fourier Series

Introduction to the Fourier Series

Commentary:
It's hard to imagine that these activities have something in common but they do, whether it's stocking supermarket shelves, using the telephone, or creating a masterpiece. There are many ways to create beautiful music but rather than trying to understand a whole orchestra let's look at a few instruments. These amateur musicians are rehearsing a piece from Bach. Usually it's possible to distinguish the violin from the flute. In fact, the composer uses these differences to create music, but why does each of these instruments make a different noise, even when they're playing the same note? This is a spectrum analyser which allows you to look at features of the note produced by each instrument, and to analyse the profile of the waveform that's produced. Again, let's simplify and look at two of the instruments separately. This flautist is playing a 'C' and here's the trace of that note. We'll store that trace and explain it in detail in a minute. Let's try again with the violin. Exactly the same note – a 'C'. Compared to the flute the sound is very different, a distinction that's often described in terms of timbre or colour. Comparing the two traces we can see peaks at the same horizontal spacing, but they're different heights. For the violin at the top the peaks die away. For the flute, despite an initial decline, there's a significant rise at the fourth peak. In fact, the regular spacing of these peaks tells us that the complex sound of the flute is the result of adding different amounts of the simplest of all periodic waveforms, the sine wave. You might find that hard to believe. The horizontal position of each spike in the trace corresponds to a frequency. The leftmost spike is the frequency of 'C'. Aside from the same periodic repeat, the sine wave seems to have little to do with the flute – but wait! A string of given length, or a column of air in the flute for that matter, vibrates at a natural frequency, a fundamental frequency or first harmonic. The left hand spike is that first harmonic. But that's not the only possible vibration mode. The string will comfortably vibrate at double that frequency, a second harmonic, and a third, and so on. The spectrum analyser tells us that in the flute sound, as well as the first harmonic, there's a contribution from the second harmonic and a third, and others as well. In fact, the flute is a mixture of all the harmonics. Start with the first harmonic, the basic sine wave. The analyser tells us how much of a contribution this makes. It needs to be scaled down. Now, take the second harmonic. That has twice the frequency, but it also has a much smaller amplitude. When we add these two harmonics the result is little different from the basic sine wave, but you'll see bigger changes later on. There isn't much of the third harmonic either. That's part of the characteristic of the flute. The second and third harmonics don't contribute as much to the sound. It's the fourth harmonic that visibly alters our waveform. Although this waveform still has the same period as the basic sine wave the presence of the harmonics alters the details, and these details for the flute will be different when we come to the violin. By the time you've completed this process for the first few harmonics you're getting a very non-sinusoidal trace. It's characteristic of the flute but it's still not exactly like the real trace. Partly that's because we've only considered the first few harmonics, but there are other peaks too. The low level noise at the bottom left of the display. They're not at the harmonic frequencies but they, too, alter the sound of the flute. We've been expressing complex musical waveforms as a sum of simpler basic sine waves. That's the job that the spectrum analyser does, but mathematically this is the branch of mathematics we call Fourier series. So what do Fourier series tell us about the violin? We start with the same basic sine wave. Remember that both instruments are playing the same note, but the contributions at each harmonic are different. When you've taken this process as far as you did with the flute, you have this, reminiscent of a plucked string, and very different from the flute. And the fact that we can see differences just from the first few harmonics, that we can ignore the higher frequencies, is the beginning of another story.