The Open University

The Rainbow Analysed

A full circle rainbow?

If you could move upwards – imagine you have access to a magic carpet – you'd see more and more of the bow, even when the sun is relatively high in the sky. This effect is sometimes seen from aeroplanes - but you don't need to be able to fly to see the full rainbow circle.
You can make a rainbow from the spray of a hose. Go up high enough and the full circle can appear……at least where there are water droplets.

Aristotle's model hinges on points over the whole cloud reflecting light back at a fixed angle, in all directions. This implies that each point sends back a *cone* of light. But Aristotle couldn't explain *why* there was a fixed angle, or why it might be different for different colours.

To improve the model and explain more of the features of the rainbow, scientists needed to look at what was happening at each individual point in the raincloud.

You'll remember that Aristotle's model implied that each point in the raincloud gives out a cone of light at a fixed angle.

Descartes considered how each of the billions of raindrops in a rain cloud might emit these cones of light. He investigated the path that a ray of light follows inside a raindrop and how it depends on the point of entry.

There's only one ray that's *not* deflected from its path, and that's the one that travels down the central axis of the drop.

But away from the central axis, watch what happens. According to Descartes' model each ray is refracted when it enters the raindrop, reflected off the back of the raindrop, and bent *again* as its refracted back out into the air.

Descartes' calculations showed that for evenly spaced light striking the raindrop, the refraction, reflection and further refraction leads to a concentration of rays, at about 42° to the incoming light. This concentration of light can be bright enough to be seen.

Of course the drop isn't just a circle, it's a sphere, and with light rays entering all around the front of the drop, the emerging light forms a 42°*cone*, with the *concentration* of light at the surface of the cone.

In his calculations, Descartes defined the path of light rays in terms of the distance away from the central axis that they hit the raindrop. This distance from the central axis is the impact parameter - we'll call it *X*.

For simplicity, we'll take the radius of the raindrop as 1, the impact parameter *X* then ranges from +1 to -1.

Using trigonometry, Descartes related the impact parameter *X* to the exit angle *Y* - that's the angle *between* the emerging ray and the incoming sun's rays.

Descartes derived a function for the angle *Y*. *K* is a constant - it's a measure of how much light bends as it passes from air into water and Descartes used a value for k of 0.748.
For each impact parameter *X*… you can plot the value of *Y*… to give this graph.

The graph is symmetrical, so in order to see the detail more clearly, let's concentrate on just one half of it. For equally spaced rays coming in from the sun, you get rays exiting at various angles. But there's a concentration of the light rays exiting the raindrop at about 42°. That's Aristotle's fixed angle.

This graph is for one colour only - yellow. But Newton discovered that white light is made up of a spectrum of different colours, which you can show by refracting sunlight through a prism.

A similar spectrum results when white light hits a raindrop. Red light, for example, will be refracted and reflected in the raindrop like this.

And the other colours are refracted slightly differently. The value of *k* is slightly different for each colour, and so you can plot a different graph for each of the different colours.

The value of *k* for red light is 0.75. That gives this graph. The value of *k* for violet light is 0.743, giving rise to a slightly different graph.

The maximum value of Y for red light is about 42°, but for violet it's only just over 40°.

Extend this picture to 3 dimensions, and you find each raindrop giving out a nest of cones of different colours, sitting one within another. They all have the same axis, but for each colour the concentration of light is at a slightly different angle. In the middle, there's no concentration of any one colour.

These different exit angles for different colours of light are the reason why colours appear in a specific order in the sky. Look at one drop as it falls through the sky.

As the drop descends, it appears as if it's changing colour.

First it seems to be red, then it changes through orange, yellow, green, blue, and so on during its fall. This is due to different cones of light reaching your eye.

High up in the cloud, the coloured cones completely *miss* your eye.

Further down, there comes a point where the bottom edge of the red cone hits your eye, and so drops in this region of the cloud appear red.

Moving further down, there's a region where drops appear green because it's now the green cone which hits your eye. And the region where the edge of the innermost violet cone reaches the eye makes the drop appear violet.

Lower down still, all the coloured cones of concentrated light rays miss your eye. But the colours you've seen come from drops directly in front of you. What about drops in other parts of the cloud?

The red arc is formed from drops where red light is visible from the *side* of the cones exiting those drops. All the drops have a red cone, but you only *see* red light from drops in these positions.

Other colours of light are visible from drops closer in towards the centre of the bow.

As in Aristotle's model, the fixed angle for each coloured cone of light coming from the individual raindrops leads to your seeing a circular arc of differently coloured bright points.