Astronomy
Geometry of the Universe

Russell Stannard
What is geometry? The geometry that describes the Universe and everything in it. Now that sounds a silly sort of question, you know, what else could it be other than straightforward commonsense Euclidean geometry? For example, if you have two parallel straight lines and you continue them on to infinity, then they would always stay the same distance apart. As for triangles, well the sum of the angles of the triangle all add up to 180 degrees and it doesn't matter what the shape of the triangle is or its size, you always get 180. As for circles, well the circumference of a circle is 2 Pi times the radius 'r' and, again, it doesn't matter what the size of the circle is, you will always get 2 Pi 'r', so that's Euclidean geometry. But there are situations where that kind of geometry doesn't work. I mean suppose, for example, you're trying to circumnavigate the globe, there you're confined to a curved surface – Euclidean geometry doesn't work there. If we've set up a geometry for this kind of surface, then obviously we need straight lines to start with and obviously to some extent there isn't a straight line there, they're all curved, but if we choose as our definition of a straight line that it's the shortest distance between two points and suppose we have two points – one there and another up there – which would be the shortest distance between those two points? And in fact it's this line here, it's a line of longitude, and in general what we find is that between any two points the shortest distance, the straight line, is the arc of a great circles, the great circle being one that lies in the plane that passes through the centre of the sphere. So all these lines of longitude are technically straight lines and so also is the Equator. So we've got our straight lines then, how about parallel straight lines? Well yes, we can take any pair of lines of longitude – let's take that one – as they cross the Equator – because we know that an angle there is 90 degrees and so is that one there – so these two lines are parallel. So let's now extend those to infinity, or at least as far as we can get on this surface and obviously what happens is that those two lines, they don't stay the same distance apart as they did before, they actually converge and intersect there at the Pole. So that's the first big difference between geometry on a sphere and geometry that is flat.

How about the triangle? Well I've drawn a triangle on here. We need three straight lines so I've chosen two lines of longitude – this one and that one – and for the third side I've chosen the Equator. Right, now, looking at the angles of that triangle, clearly this is 90 degrees – we've said that before – and so is that. Add those two together and that gives us 180, but you know we've still got to add in that angle there, so whatever that might be it's obviously going to end up more than 180 degrees – the sum of the angles of that triangle. In fact that is true not only of this triangle but of any triangle that we draw there; it always comes out to be more than 180 degrees. Alright, if you have a very tiny triangle, an absolute sort of tinsy-winsy little triangle such that it covers an area which is more or less flat then the sum will come out to be roughly 180 degrees but it will still, strictly speaking, be a little bit more.

Right, thirdly, what about a circle, well, let me draw a circle on here and to make things simple let me just draw it to be the same radius as we had here so we'll take the pole as being the centre of the circle and I swing this round, and lo and behold the circle happens to be the Equator alright so the Equator is the circle of radius 'r' that we had before. So now let's just compare the circumference of the flat circle of radius 'r' with the one on this surface. Well, as you see, there's no comparison at all, the circumference of the Equator is obviously much smaller than we've got here and of course it has to be when you come to think about it because you know this radius is curved. And that's what one finds with all circles that we draw on this surface that the circumference comes out to be less than 2 Pi 'r' because remember this is what is 2 Pi 'r'. So that is what we call spherical geometry and, as you see, it's very different from flat Euclidean geometry. But even that doesn't exhaust the possibilities. Now suppose, for example, we had a surface like this, the kind that a horse-flea perhaps would have to learn how to negotiate – a saddle shape or, as we sometimes call it, a hyperbolic shape. Well, again, the first thing we have to do is to identify straight lines, and again we use the convention that it's the shortest distance between two points, so that gives us some straight lines. How about parallel straight lines? Well, there are a couple down here – that one, and that one there – they are parallel - and if we follow that through, well you see that again they don't stay the same distance and neither do they intersect with each other as they did on the sphere – they don't diverge. So that's a big difference from the sphere. Triangles – yes, I've got a triangle over there and as you can probably judge without measuring, the sum of the angles of that triangle are going to turn out to be less than the 180 degrees. Alright, now you might think that I'm cheating a bit because you might think, well, he's only actually only got one decent straight line there – the other two are curved, he's deliberately curved them in order to make the sum come out less than 180 degrees, but actually that's just a trick of the perspective. If I swing things round a bit you'll find that that straight line begins to look curved and the line on the left now begins to be the one that looks like a straight line, so what I'm saying is that you must be very careful not to go by appearances, and stick by the convention that these lines are the shortest distance between two points. And so okay, with that triangle and any other triangle that I've drawn here, the sum comes out to be less than 180 degrees, whereas on the sphere it was more than 180.

Finally, how about a circle? Well let me again draw a circle on our curved surface with the same radius as we had originally and we'll take the point here in the middle as being our central point and well, as you see, I've already drawn that circle in there. This is the circle, and so what I have to do now is compare that new circle with our flat one. If I just put that in there I think you'll agree that's a pretty good fit that side, so let's swing the whole thing round and see how it fits on the other side. Huh, as you see, it doesn't. There really is a big difference there. And again I'm not cheating. If I take our radius you'll see that it really is a circle of radius 'r' again and, as you see, it turns out this time to be very much larger than the circumference of the flat circle. So what this means is that on this surface the circumference of a circle is greater than 2 Pi 'r', whereas there it was less.

Right, now, you might be saying well – why bother with all these different kinds of geometry? Well, the point is this – that it could well be that Euclidean geometry does not apply to the Universe. You know I glibly started off by saying that these two lines, if stretched to infinity, would remain the same distance, but how do we know that you know? We've never actually ever stretched lines to infinity as far as we know if we actually did the experiment we might find that they intersected like they did on the sphere or diverged as they did on that hyperbolic surface. As for triangles, well yes, if I measured these angles I'd find that they come to 180 degrees, as precisely as I can measure it but this is a small triangle, you know, compared to the size of the Universe. And we saw on the sphere that if you had a very tiny triangle, then even there you would get roughly 180 degrees. What would happen if we had a huge and absolutely gigantic triangle? Suppose, for example, that's us on our galaxy and we had another observer on a different galaxy and yet a third one over there on another galaxy – and suppose each of us took lines of sight to the other two galaxies and then measured the angle between the two, and then somehow we pooled our results and added the three angles together – what would we get? Would it be 180 or would it be something somewhat larger as we got on the sphere, or smaller as on the saddle? We don't know - we haven't done it. And as for the circle – okay, it's roughly as close as one can say 2 Pi 'r' but, again, if we had an absolutely huge circle what would we get then? We don't know. In fact, we do not know what the geometry of the Universe is. It's one of the big problems of cosmology.