The Open University

60-Second Adventures in Thought

Hilbert's Infinite Hotel

A Grand Hotel with an infinite number of rooms – and an infinite number of guests in those rooms.

That was the idea of German mathematician David Hilbert, friend of Albert Einstein and enemy of chambermaids the world over.

To challenge our ideas about infinity he asked what happens if someone new comes along looking for a place to stay?

Hilbert's answer is to make each guest shift along one room – the guest in room one moves to room two and so on – so the new guest would have a space in room one. And the guest book would have an infinite number of complaints.

But what about when a coach containing an infinite number of new guests pulls up – surely he can't accommodate all of them?

Hilbert frees up an infinite number of rooms by asking the guests to move to the room number, which is double their current one, leaving the infinitely many odd numbers free.

Easy for the guest in room one. Not so easy for the man in room eight million, six hundred thousand, five hundred and ninetyseven.

Hilbert's paradox has fascinated mathematicians, physicists and philosophers – even theologians.

And they all agree you should get down early for breakfast.