Exploring mathematics: a powerful tool

Isaac Newton's input

Anna Southgate
Wallis's book had a great influence on one the creators of calculus, Isaac Newton.

Newton was born in 1642, more than a decade after Fermat had discovered how to find areas and gradients. He came to study here at Trinity College, Cambridge, in 1661.

Newton soon began studying the works of leading mathematicians, including Descartes and the Dutchman Christiaan Huygens. Lectures by Isaac Barrow the professor of mathematics here at that time, taught Newton about optics, and Fermat's methods of finding gradients. But life in seventeenth century England was sometimes precarious.

1665 to 1666 was the time of the great plague in England, and Trinity college was closed. So Newton returned home to Lincolnshire, which proved to be his most productive time, because it was there he made a remarkable discovery about the binomial theorem, and link this, to both the area and gradient problems.

Jeremy Gray
Newton studied Wallis's book on the arithmetic of infinites, and this lead him to a novel idea - expressing answers to problems as infinite series in powers of x. Wallis had wanted to find the area under this curve, of the equation y=(1-x2)½, all the way from x = 0 to x = 1. Newton found he could do much better. He could find the area from x = 0, to any value of say this one.

He expressed the answer as an infinite series in powers of x, which is a remarkable thing to do. At first his methods were rather like those of Wallis's actually, clever guess work. But he soon found he could make systematic sense of the infinite series that arose, and in this way he discovered the general binomial theorem.

The form of the binomial theorem had been known for a long time for integer values of n. You get an expression like this. A finite number of terms, the value of n goes down by one each time, and the number of terms and the denominator goes up by one each time.

What Newton discovered was a different expression for arbitrary values of exponent, for example a half here. You still find that the exponent goes down by one each time, that the number of factorial terms (the terms in the bottom) increase by one each time, but now the whole expression is infinite, it never stops.

Anna Southgate
So why are fractional powers so important?

Jeremy Gray
Well Newton used his method of infinite series to solve area and gradient problems for all sorts of curves, including curves with fractional exponents.

In the course of doing this work he discovered something very important. If you take a curve like this with a fractional exponent like y = x½, Newton found that he could investigate the area under this curve from 0 to any value of x, and he found in this case that it was 2/3x3/2 . And now because he was happy with fractional exponents, he found that he could investigate the rate of change of the area curve, and he found that the rate of change of the area curve at x, was equal to the height of the original curve, y = x½. So he had a way of going from area problems to gradient problems, and back, and this is what we call the fundamental theorem for calculus.

Anna Southgate
So in effect, Newton found areas by looking at rates of change?

Jeremy Gray
Oh absolutely, and that's very important because finding areas is difficult, and finding rates of change is easy. So Newton could solve hard problems, area problems, by finding that there were solutions to other problems, easy problems, that were gradient problems.

Anna Southgate
So how were Newton's results recorded?

Jeremy Gray
Well there was quite a lot of secrecy, but Newton did communicate his results to friends, and in 1676 he even wrote a letter to Leibniz, a long letter describing what he'd done, but the key results that he'd found, he only gave in the form of acronyms, forty letter acronyms, which obviously didn't tell Leibniz anything, and actually that's the point because, what Newton is doing then, is signalling that he's got priority in these discoveries, without letting on what they are.

Then a bit later Wallis started to circulate some of the things that Newton had discovered, and then finally of course in 1687, Newton really does publish.

Jeremy Gray
He published his great book the Principia Mathematica, which is the book that applies geometry
to the study of the solar system, and lots of other topics in applied mathematics.

So, now Newton was thinking in terms of fluctions and fluents for the calculus, and in this book in fact, he'd moved on in fluctions and fluents to first and last ratios, which are a limited concept much like we have in the calculus for this day.

But actually the Principia isn't written in the language of the calculus. There's little bits of calculus in it, it's really a geometry book.

Anna Southgate
Unfortunately, Newton's writing was full of difficult concepts, he also used a rather specialised notation.