Exploring mathematics: a powerful tool

New insights emerge

Anna Southgate
However, a clearer version of these new ideas emerged from elsewhere in Europe. Niels Jahnke is a Maths Historian from the University of Essen in Germany.

Can you tell us a little bit about Leibniz's background?

Niels Jahnke
Well, he was born in 1646, and he went to the University of Leipzig in 1661.

He was talented in many areas - in philosophy, theology, languages, law and mathematics. And at the age of twenty he was offered a professorship in…at the University of Aldorf, but he refused it and preferred to become a diplomat.

Anna Southgate
And what got him interested in these problems in the first place?

Niels Jahnke
Well it was in a diplomatic mission when he came to Paris in 1672 and, in Paris he met Christiaan Huygens, and Huygens was a famous physicist of his time, and also a very very good mathematician, and Leibniz was eager to learn mathematics from him.

And so, Huygens posed him problems, and he recommended him works he should study by other mathematicians, among them, above all, the works of Pascal. And then after hard work he…in 1675 he devised his own algorithm for determining tangents to curves, and he got the insight in the inverse nature between this problem of determining tangents, and the problem of the area under a curve.

Anna Southgate
Niels explained Leibniz's insight to me.

Niels Jahnke
He built a rectilinear model for analysing curved lines. So he considered every curved line as a polygon with infinitely many sides, which are infinitely small.

So look for instance, here we have two points, and we connect these two points by line segment, and this line segment is infinitely small. And then Okay we have here the, y co-ordinate of the two points, and we get here the x co-ordinates, and now we can say what a tangent is. A tangent line is simply an extension of this infinitely small segment. So we have here this tangent line, and Leibniz, as others at the time, called this, from here to here, as a sub-tangent.

And so we have here the ordinate, and the x co-ordinate, and then, for this second point, we have another y co-ordinate and x co-ordinate, and the difference between these two ordinates is the difference dy, and the difference between the two x co-ordinates is the difference dx. So in fact, these are really differences.

And now it's completely easy to determine the tangent line. There are two triangles here which are similar. This infinitely small triangle, and this finer triangle up here. And so we have the proportion $dy/dx = y/t$.

Anna Southgate
And Niels, what about the area under the curve?

Niels Jahnke
Yes, for this problem of the area under the curve, he again applied his rectilinear model. So we have the sequence of points on the curve, infinitely many points, and we have all these co-ordinates. And then we introduce the function z, quantity z, and z designates the area under the curve. And what Leibniz does is to calculate the differential difference of this z.

So we have here, the x co-ordinate x, and the y co-ordinate, and the differential is the difference between the area between the origin and this point, minus the area between the origin and this point, and so the differential is a shaded strip.
And we have here the difference between these two co-ordinates is dx, and up here, dy. And dz is exactly this difference between the two areas, it's exactly this shaded strip. And we can calculate this, as dz = ydx.

But by writing this equation, we have neglected this infinitely small triangle up here, but it can be easily shown that it is really infinitely smaller than this rectangle made up of y and dx. So this is correct within this calculus. And now, you take the sum over all this dz, and this gives the area. So the sum is simply our z, and this is equal to the sum over all ydx, and this integral sign here, as we are used to, is derived from the normal S. And so we have derived the fundamental theorem.

Anna Southgate
Leibniz invented the integral sign that's still used today. It's a long stretched S, because he saw the process as summing.

Niels Jahnke
And in fact he invented also this letter d for the process of taking differences, differentiation, and this comes the name differential calculus, and in fact this notation made it very easy to calculate. And so, very fast, he arrived at rules for differentiation, for instance a product rule and the quotient rule, and at the same time he had some communication with Newton, and they exchanged their results, and they realised that they both were able to derive theories for sign, for logarithm, and other similar results.

But Leibniz was very much aware that it was important to publish his results and therefore he published his account to the differential calculus in the paper in 1684 in the Acta Eruditorum. This was a journal he had founded together with others some years ago, and he continued publishing other papers in this journal in the years to follow.