Exploring mathematics: a powerful tool

A vibrating lake

Carmen Pryce:
Here's Lake Wakatipu in the South Island of New Zealand which is famous for a noticeable rise and fall in the surface of the water every few minutes. A mathematical model of this natural phenomenon is being developed at the University of Otago, and to show you how a model like this can quickly get very complex let's try to describe the rise and fall using a very simplified version of the maths that the model has to cope with. First, what's seen to happen in the lake? Well in fact, it's not unusual for sections of any lake to rhythmically rise and fall but, uncommonly, in some places the surface level of Lake Wakatipu can change by several centimetres in just five minutes. This regular rise and fall is known as a seiche and is often driven by the weather conditions. For Lake Wakatipu the atmospheric conditions around the mountains of New Zealand's South Island seem to be responsible for the unusually noticeable seiche in this zig-zag shaped lake. An instrument which is able to make recordings of how the water pressure above it changes can be placed at different points all around the lake. The pressure readings can then be converted into data about the changing depth of the water over time. This meter is being retrieved by Dion Burns, an applied mathematician, who's been studying the seiche on Lake Wakatipu.

Dion Burns:
Seiches occur when pressure or forces apply to a lake in an uneven fashion. Causes can be like a flood discharge from a river, perhaps seismic activity, but more commonly pressure fluctuations, wind gusts such as here at Lake Wakatipu.

Carmen Pryce:
Once the water meter has been recovered the data is downloaded into a computer which is able to smooth out minor changes such as local surface turbulence from wind-blown waves, and reveal data about the general rise and fall of the lake.

Dion Burns:
Here at Lake Wakatipu the seiche is caused by two very high mountain ranges on either, one on either side of the lake, and the wind that is channelled between them pushes on the surface of the lake, it happens in a very regular fashion. With most lakes we see the seiche occur with a rise and fall, and as one end rises the other end falls.

Carmen Pryce:
Very broadly speaking, and greatly exaggerating the motion, you might expect the lake to act like this. You can easily get other oscillations though, and to give you something of an idea of how systems can vibrate, here's a set-up in which a length of string is agitated at one end by a motor. At a certain frequency the string vibrates in this pattern, it's called a standing wave. The points where the string seems to hardly rise or fall are called nodes. By increasing the vibrations of the motor you get this second pattern for the next standing wave, and an extra node. Higher frequencies of the motor give further standing waves and more nodes. But how does all this apply to the lake?

Dion Burns, University of Otago:
Here on Lake Wakatipu we see instead the second pattern occurring whereas the edges of the lake rise, the centre of the lake falls, and as the centre of the lake rises, the edges fall.

Carmen Pryce:
For the vibrations in the lake a second pattern will have just these two nodes. Unlike the string the lake isn't tied at its ends and so you wouldn't expect nodes at either end. So the lake then vibrates in something like this manner. But there will also be other patterns to some

extent or other happening. The next vibration pattern has these nodes and behaves something like this. More vibration patterns can and will occur, as they do with the string. But remember, each of these patterns would be happening at the same time, to some extent or other. And a mathematical model has to unpick each pattern in order to describe the rise and fall of any one part of the lake. So even this simplified model can begin to get very complicated indeed. And making simplifying assumptions is really the only way of mathematically modelling natural world systems of this sort.

Dion Burns:
What we do on such a complex phenomenon is we make a number of assumptions about it. We assume, for example, constant depth, we assume the geometry of the lake to be fairly simple, and we can then apply our mathematical models to those simplified versions. We then solve the problem and we later return to the lake and take some experimental data. If our experimental data then agrees with our theoretical result, we can start to refine and remove some of the assumptions and from there get a more and more accurate account of the lake.