Exploring mathematics: a powerful tool

Fermat's ideas

Anna Southgate
Jeanne took me through some of Fermat's ideas for finding maximum and minimum points.

Jeanne Peiffer
Oh let me show you in the…Fermat presented an algorithm, a mechanical procedure, without no justification. So we look at an example which Fermat also presented.

Here you have a curve. y = 2x2 – x3, and that curve may have a maximum at x. x + A then is a nearby value, then the two values of the…of the y is here, are nearly equal, as you can see on the picture. So, Fermat is putting the two expressions equal to each other. So we have this equation here.

Then Fermat is doing some manipulation, and at some point, he is neglecting all the terms in A, because, A is infinitely small, and so he gets this equation, and the result. You have a maximum of x = 4/3.

Anna Southgate
So could Fermat also find tangents?

Jeanne Peiffer
Oh, I would show you again an example. Fermat considers a parabola, which is here with an axis horizontal, and he wants to construct a tangent to the parabola at point B, which is here. In the Euclidian way of constructing straight lines, you need a second point. So Fermat is trying to find the intersection E with the horizontal axis. So he has to find the length (E,C). Now what does he do? He considers a second point F on the tangent, which is very close to B, and which is also very close to the parabola here. The distance between the tangent, and the parabola, is very small. So again, Fermat can dequate the two values for y. Then he does some manipulation again, and he can apply exactly the same algorithm we saw before, and he will find the answer, that the sub-tangent (E,C) is twice the length of (D,C), in the case of the parabola.

Anna Southgate
So he knew how to find tangents?

Jeanne Peiffer
Yes, but he also developed the remarkable method of calculating the area under the curve y = xn .

Here is dividing the x axis by a certain number of points, x, ex, e2x and so on, e being less than one. Then he's constructing rectangles on these points, x, ex and so on, and he can calculate the areas of these rectangles.

The areas of all these rectangles form an infinite geometric sequence, and Fermat was able to calculate that sum. Then he sets e = 1, and all these rectangles are infinitely thin. And the sum of these infinitely thin rectangles is then equal to
xn+1 /n+1.

Anna Southgate
Does Fermat make any connection between the gradient problem and the area problem?

Jeanne Peiffer

No, he couldn't really do so, because he was asking geometrical questions. He was looking for construction of the tangent on the one hand, and constructing an area on the other hand.

To construct the tangent, he had to search for a second point to be able to trace the line, and to construct the area under the curve, he constructed a sequence of rectangles. So even if his methods are algebraic, the questions he asked were geometric, so he couldn't really make the connection.