Rotating Bodies and Angular Momentum
Spinning on Ice

Narrator:
Batons and maces are not the only bodies that rotate. Sometimes it can be the human body itself and that's more flexible. It's easier to move on the ice as there's less friction, but the skaters still need torques to make them turn. Skaters often use spins as part of their routines. As with the baton and mace, angular momentum is conserved. Yet, skaters can change their rate of rotation! It needs a closer look to see how it's done.

Tony Barron:
You start the spin with what we call a wind up which is basically progressing backwards round a circle, and then stepping back into the centre of the circle, erm with a quite a strong push, so that you then start your rotation approximately in the centre of the circle. You change your body position by bringing your leg in and your arms in to increase your rotational speed, as you pull yourself in closer to your spinning axis.

Judy:
So to begin a spin the skater needs a torque. But what happens then?

Dave Cobner:
Well, the effect of the torque, Judy, is actually determined by the distribution of body mass. The body is made up of a number of segments, each having different mass. The lower leg, upper leg, etcetera, have different masses and they move at different speeds. Once you change the relative position of those body parts it changes the rate of rotation or your angular velocity.

Judy:
So the angular velocity changes but that doesn't mean that the angular momentum changes?

Dave Cobner:
Well, that's correct. I mean, we've seen similar examples - if you look at someone trying to spin their chair.

Narrator:
To start this rotation you still need a torque; such as somebody pushing the back of the chair. The chair starts rotating at a constant angular velocity. But bringing the arms closer to the body speeds up the rotation and extending the arms slows it down. Total angular momentum of a body is the sum of the angular momentum of all its parts - the body, and the arms, and the legs.

Dave Cobner:
As you can see from the examples we have on the screen, the young girl with her arms extended is distributing her body mass further away from the axis of rotation, so she rotates quite slowly. On the left you see an example of her with her arms brought in close to her body which will speed up the rate of rotation.

Judy:
So how do we handle that quantitatively?

Dave Cobner:
Well, quantitatively is a term used for the distribution of body mass is 'the moment of inertia', which is in effect a measure of the body's reluctance to rotate and in effect angular momentum is a product of the body's moment of inertia and its angular velocity.

Narrator:
Angular momentum l is the product of the moment of inertia I and the angular velocity ω, which measures the rate of rotation. With the arms out the moment of inertia increases and so in order for the angular momentum to be conserved the angular velocity must decrease.

Judy:
So, once the skater has initiated the spin she can speed up her rate of rotation by bringing her arms in close to the axis of rotation - so reducing her moment of inertia. And if she puts her arms out again, that will increase her moment of inertia and so slow her down.

Dave Cobner:
Yeah, that's correct. I mean, obviously there's also some friction with the ice and some air resistance that will tend to slow her down. But the bulk of her slowing down is going to be caused by a redistribution of body mass and an opposite torque to bring her out of the spin.

Narrator:
There are lots of attractions at the fair. All sorts of side shows to keep us amused. We've come back here to look at a variation on a fairground attraction – the Roll a Tin.

Judy:
(TO CHILDREN) You want the, you want the liquid one in the middle, you go for the frozen one. (CHILDREN RESPONDING TOGETHER)

Narrator:
The right hand can is empty. The middle can is full of liquid. And this can is frozen solid. All of the cans are identical in size and shape. So the only difference is the state of the content.

Judy:
Who thinks the empty one's going to win?

Child:
I do.

Judy:
Who thinks the liquid's going to win?

Child:
Me.

Judy:
Who's the solid one?

Children:
Yeah.

Narrator:
Which one do you think will win?

Judy:
Everybody out of the way… ready, steady, go.

Narrator:
The empty can gets left far behind. Why is this?

Children:
Yeah.

Judy:
The middle one won.

Narrator:
When the can is empty, most of its mass is around the outside.
Looking inside a full can reveals a different picture.
Now a lot of the mass is inside, closer to the axis of rotation, and it’s able to roll faster.

Dave Cobner:
Well there are clear similarities there, Judy, with the examples we saw earlier on of the young girl sitting in the chair and the skater spinning on the ice. When their distribution of mass is greatest, when their hands, and arms and legs are spread away from the axis of rotation, then they tend to rotate slowly, and the arms and legs and the mass is moving closer to the axis of rotation, then you see a fast rotating body.

Judy:
So that’s why the empty can goes slower?

Dave Cobner:
That’s correct.

Judy:
Ready, steady, go…

Narrator:
But that doesn't tell us why the solid can is slower than the liquid can. To understand that we need to look inside the cans once more. The reason the liquid can beats the solid is that not all the liquid will rotate with the can, and so the outside of the can moves at a slightly faster angular velocity than the inside to compensate. With the Roll a Tin, the rotation of the can is combined with its motion under gravity, down the slope.