



## Mathematical modelling

### *Modelling pollution*

#### **Susan Rae**

And the kind of modelling that's needed to make *quantitative* predictions is *mathematical modelling*. The kind of modelling where features are represented by mathematical variables and parameters; and where relationships can often be described by equations.

Bill Richardson has strong views on when mathematical modelling is needed.

#### **Bill Richardson**

Why it's important to do mathematical modelling? I am asked that a lot. We spend a lot of money on computers and of course our salaries, and a lot of other things, to do what we do. And my first response to that is that you don't always have to do mathematical modelling to solve water pollution problems. In fact, I mean it's maybe the last thing you would do.

The Cuyahoga River in Cleveland, the Rouge River in Detroit, the oil slicks and the gasoline on it that floated to the surface caught fire from sparks from a welder's torch.

First you have to take care of those obvious problems.

Now, once you've taken care of the obvious problems, what remains, what are...what are the issues?

What happens is, decisions have to be made that are much more subtle, that involve the gross pollution problems. It becomes a matter of degree now. How much of these toxic substances, should we ban DDT – I mean that was a big decision back in the early seventies, do we ban PCBs? These were important agricultural and industrial chemicals that had great benefits to solving other kinds of problems. So by banning those we might even create a worse public health problem.

So once you start asking these economic and social public health problems, then you need a scientific basis for making those decisions. They might have to go to court. The judge may have...have to have you prove, or show scientifically, that if we spend a certain amount of money, or if we do make a certain decision, ban a certain chemical, that we in fact will get a return, there will be a benefit to what we do.

So in order to do that, we have to understand how these chemicals behave in the environment. What the cause and effect relationships are between what is coming in from the various sources, industry, municipalities, run off from agriculture, run off from urban lands, atmospheric deposition.

We have to understand how to understand how those impact...those loads impact the environment, how the chemicals move around, transport, what their fate is, how they bio-accumulate into the fish, how they can work their way up into the human body.

And the only way that we know how to do that is to develop mathematical models that define these processes. And once we have the mathematical models and that basic understanding, the next question is: how valid are these models?

**Susan Rae**

To help validate its models, the EPA has its own survey ship, the Lake Guardian. This cruises the lakes throughout the summer months, using a range of specialist equipment to take samples.

Each mission is carefully planned. The samples aren't taken at random. The map reference of each sample source is chosen deliberately.

These samples come from Grand Traverse Bay in the north-east of Lake Michigan. Taking samples from different places allows the *mixing* of the lake to be monitored.

And water samples can come from different levels in the lake. The bottles on this device have special mechanisms so that they open and close at predetermined depths.

These samples help to monitor the mixing between different *layers* of water.

**Bob Thomann**

Water quality is measured by going out in a boat, taking a sample of the water, bringing it back to the laboratory and characterising the chemical and biological variables in the water.

For example, the oxygen level in the water, the nitrogen, the phosphorous level, whether there are any toxic chemicals, PCBs, the heavy metals. So water quality would be characterised by a suite of parameters that describe the chemical, and physical, and biological integrity of the water.

We also measure water quality by examining the sediment, because the sediment in the water are clearly interactive. And in the same way, you go and you drop a dredge into the bottom of the sediment and you bring up the sediment, bring it back to the laboratory and measure it.

**Susan Rae**

But however simple or complicated the model, there'll always be a need to measure some of the basic parameters involved – for instance, the *volume* of the lake. Knowing the volume is the only way we can relate the mass of pollution that's been deposited to the eventual concentration that can be measured.

**Bill Richardson**

Well the volume is a very important parameter. Without that we don't know how long the pollutant will be retained in the lake or how long the water will be retained. So we need to know the volume. And basically there's a very simple procedure. We have information on the bathymetry of the lake that they've collected over many years.

The lake levels are measured, so we know fairly accurately the depth of the water at pretty much every square kilometre in the lake. And basically we just add up these segments of water – knowing their area and the depth – we add up all these segments for the entire lake and that becomes the total volume of the lake.

**Susan Rae**

As well as the volume of the water, another quantity that has to be investigated is the water *flow-rate*. For pollutants that don't degrade flow of water is the only way of clearing them from the lake.

The mathematical models used for pollution in the Great Lakes will obviously get very complicated as more and more features are included. But there's a role for simple models too.

**Bill Richardson**

A lot of scientific modellers, people who are coming from a...from an academic point of view, really start complicated and try to include everything, and try to include as much as possible because they're scientists and they want to understand the whole thing.

But what I've learned over the years, and what...one of the basic engineering principles that I go by, that I think Bob Thomann actually taught me, was to start simple and work toward the complex. And the reason you do that – now there are several reasons why you do that. One is that these models that end up being very complex actually build, you build your building blocks, and one thing builds on another. And if...as you build this you need to gain understanding of basic simple things, and then build the next level of complexity, and then the next level of complexity, and then finally you have the whole thing. But along the way you have much more confidence in what you've done.

### **Bob Thomann**

Well a simple model is useful because it's simple. It allows a rapid computation on a global type of question. For example, for the Great Lakes the global type of question would be to first approximation how long would it take this lake to flush out the concentration starting at a given level? And when you first do that kind of calculation you don't need a lot of complicated features because the principal mechanism is the volume of the lake, and the flow through the lake, and the degradation process of the chemical. Those are gonna be operative even in the most complicated models.

So the simple model is very helpful for those initial first estimates. In fact since modelling is a type of art form, it's very important to start with the more simple calculations, and sometimes a major portion of the information that the decision-makers really want is accomplished from simple models.