Maths as others see it

Spherical gemometry

NARRATOR
Cameron Balloons, in Bristol, build the sort of balloons that you'd use for a nice flight over the Peak District in Derbyshire, but they also have expertise in building balloons that could take you right around the world.

NEWSREADER (MOIRA STEWART)
A British pilot and his Swiss team-mate have become the first balloonists to fly around the world non-stop. Brian Jones and Bertrand Picard entered the record books this morning, in their Breitling Orbiter 3 balloon. Their journey started 20 days ago in Switzerland; 26 602 miles later they made aviation history, crossing the finishing line in North Africa.

ALAN NOBLE
A Breitling Orbiter round-the-world balloon isn't like the hot-air balloon you'd go for a nice, serene flight over the countryside in. To start with, it doesn't use hot air to provide its lift; it uses helium gas.The balloon has an outer skin of – silver – it's an insulating skin. It has the gas cell containing the helium here, below that is a hot air cone, and at the very bottom is the gondola with its burners and fuel system.The gondola carried some very sophisticated communications and navigation equipment. In fact it was all controlled from simple laptop computers like this one. But it meant that from the control centre, in Geneva, I could send messages to the crew, information from two of the three global positioning systems on board was fed to the laptop, and that would come down to us in the half-hourly messages – which meant I could look at a screen in the control centre, and I knew where they were to within fifty metres during their entire flight around the globe.

NARRATOR
For navigating short trips, like the ones our two balloons made, pilots use flat charts and Pythagorean geometry. But what about larger-scale journeys? A journey round the world requires us to take into account the curvature of the surface. Here, Pythagorean geometry is no longer applicable. A different geometry is required. This is the world of spherical geometry, where the shortest distance between any two points over the surface is no longer a straight line but is in fact an arc of a great circle. A great circle is centred on the centre of the sphere and lies on a plane that bisects the sphere. The system of identifying points on the earth's surface using degrees of longitude and latitude utilises great circles. All lines of longitude are great circles, that is, they cut the earth into two equal hemispheres. Now, the equator is a great circle, but other lines of latitude are not great circles. Although the plane of this line of latitude is a circle, the radius of this circle is smaller than that of the earth. So it cannot be a great circle.

There's another difference between flat Pythagorean geometry and spherical geometry. Consider a triangle drawn on the surface of the earth. Let's say you travel from the North Pole due south on the Greenwich meridian – that's zero degrees longitude – till you meet the equator. You then turn 90 degrees left and travel on until you meet the 90-degrees-east line of longitude. Again, turn 90 degrees left and continue on till you're back at the North Pole. Now, this is a triangle with three right angles, a total of 270 degrees – not the 180-degree sum you'd expect a flat triangle to have. However, the internal angular sum depends upon the size of the triangle. And, as you might expect, the sum of the internal angles of a very small triangle is 180 degrees. So, to navigate round the world, the Breitling team clearly needed to understand spherical geometry, as well as navigating by the winds, as used in the Peak District balloon flights which you saw earlier.

ALAN NOBLE
Navigating a balloon is quite interesting because, of course, it doesn't have a rudder or a tiller. The only way you can change direction is to get into a different wind current. And so in the control centre we'd talk with meteorologists, and they would give us forecasts for up to five days ahead as to where the balloon should be going. When the balloon lifted off from Switzerland, the winds were taking it to the south-west – not the direction you want to go for an around-the-world flight. But the meteorologists had been very confident that if we could get the balloon down to north Africa, then it would change direction when it picked up the jet stream winds, and it would go east, and they were absolutely right. Once the balloon had landed in Egypt I was pretty relieved, and that was really the end of my job. But there are rules governing flying a balloon around the world, if you want to claim records, and so we had to collate all the data, submit it to the mathematicians, and it was then their problem to work out just how many records we'd broken, and by how much.

NARRATOR
Don Cameron is one such mathematician.

DON CAMERON
The rules for the round-the-world flight were set by the FAI, the Fédération Aéronautique Internationale, and their problem was that for the shorter balloon flights, they had used only the great-circle distance; they weren't interested in the wiggles of the balloon's flight, just the distance from take-off to landing. And as balloons started to go more than half way round the world, clearly the rules had to change. If you get right round then you'd get a very small distance by that rule. The notion of waypoints was introduced. They have to be substantial distances, so it's only really for the very long-distance flights that they're used, and the rule is the distance between way points mustn't be shorter than half an earth's radius; the average mustn't be shorter than a complete earth's radius. It's a sequence of little great-circle distances which are taken, and this is done by using the mathematical formula for great-circle distances and then simply adding them up around the world. I've done a programme for this which shows a chart on the screen, which shows the basic track and then also shows all the waypoints, finding their way around. The list of latitude and longitudes form this rather wiggly line on the screen; this is divided into legs. The first leg is here, coming from Switzerland to a point in Saudi Arabia, and then subsequent legs going through Asia, the middle of the Pacific Ocean here, Central America, the coast of Africa, and finally landing in Egypt.
The FAI rule for distance measurement is based on an assumption of a spherical earth, and for that reason it's a spherical calculation – I'm using the formula for a great-circle distance on the surface of the sphere.

NARRATOR
Global-positioning-satellite data was used to calculate the distances between the waypoints. But Don's programme is one of several that are available.

SUE TATFORD
There is a commercial package available for measuring distances between two points on the place of the earth, by the great-circle route. The way points or the position reports from the balloon can be plotted, and their route can be shown: into Africa, across India, east Pakistan, Burma, China, and out into the Pacific Ocean; across Mexico, the Caribbean, the Atlantic, following a very similar route to their way out, and landing in Egypt. The software package also shows distances between two points via the great circle route. And we can go from Kito, across to Entebbe, and it gives a distance, a great circle, of 12 292 kilometres.

COMMENTARY
But how can this package and Don Cameron's programme calculate such distances?
It's easier for places like Quito, in Equador, and Entebbe, in Uganda, that lie on the equator, because the equator is a great circle.You just need to know the angle subtended by the arc that separates their two longitudes, and the radius of the earth. And then the distance between the two cities can be calculated using the formula l equals R theta, where theta is measured in radians.Theta is the difference in longitudes, being careful to account for the zero meridian. Quito is 78 point five degrees west, and Entebbe is 32 point five degrees east. So the total angle is 111 degrees.

This angle can be converted to radians using the formula, one degree equals pi divided by 180 radians.Thus theta is 111 pi divided by 180 radians. Now, multiply theta by the radius of the earth, to get the numerical value for the great-circle distance.Although the earth isn’t a perfect sphere, we can use an average value of 6371 kilometres. So, the distance between Quito and Entebbe is 12 300 kilometres, to the nearest hundred kilometres.